Ion acceleration in plasmas

Lecture 5. Laser absorption and energy transfer in plasmas

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1. Short Pulse Interaction Regime

2. Ionization

3. Collisional Absorption

4. Collisionless Absorption

5. Hot Electron Generation
Short Pulse Interaction Regime

Ionization of solid by laser field in first few cycles rapidly creates a surface plasma layer with a density many times the critical density \( n_c \)
\[
( n_c = \omega^2 \frac{m_e}{4\pi e^2} )
\]

In practical units: \( n_c = 1.1 \times 10^{21} (\lambda/\mu m)^{-2} \text{ cm}^{-3} \)

For example: Al (aluminum) has 3 valence electrons; 6 more can be created by few hundred eV.

Electron density,
\[
n_e = Z^*n_i = \frac{Z^*N_A\rho}{A}
\]  (5.1)

For, effective ion charge: \( Z^* = 9 \)
Atomic number: \( A = 26 \)
Avogadro number: \( N_A = 6.02 \times 10^{23} \)
Mass density: \( \rho = \rho_{solid} = 1.9 g\text{cm}^{-3} \)
Electron density: \( n_e = 4 \times 10^{23} \text{ cm}^{-3} \)
Density contrast (\( \propto 1 \mu m \) laser): \( \frac{n_e}{\rho} = 400. \)
Heating

Depending on the intensity of laser, target is heated by electron-ion collisions to 10 s or 100 s of eV.

The plasma pressure created during the heating process causes ion blow-off (ablation) at the sound speed:

\[
c_s = \left( \frac{Z^* k_B T_e}{m_i} \right)^{1/2} \approx 3.1 \times 10^7 \left( \frac{T_e}{\text{keV}} \right)^{1/2} \left( \frac{Z^*}{A} \right)^{1/2} \frac{\text{cm}}{s},
\]

\[ (5.2) \]

- \( k_B \) - Boltzmann constant,
- \( T_e \) - Electron temperature,
- \( m_i \) - Ion mass.

The density profile created is exponential over scalelength due to ion ablation:

\[
L = c_s \tau_L \approx 3 \left( \frac{T_e}{\text{keV}} \right)^{1/2} \left( \frac{Z^*}{A} \right)^{1/2} \tau_{fs} \text{Å},
\]

\[ (5.3) \]
Ionization (Relevance)

For multi-electrons atoms, ionization degree, $Z^*$, needed for basic plasma properties like the electron density, equation of state, transport coefficients.

→ High density, Optically Thick Plasma: Radiative and Absorption mechanism balanced

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Local Thermal Equilibrium (LTE) reached.

→ Short Pulses: Optically Thin Plasma: Radiation Disappears

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Which span many orders of magnitude in density and temperature all at once (non-LTE).
Collisional Absorption

→ Long pulse (ps-ns) → density scalelength $L/\lambda \gg 1$ (e.g. 10-100). Laser field mainly absorbed in underdense region via Inverse Bremsstrahlung (IB).

→ **Sub-picosecond, low intensities** ($I < 10^{15} \text{ W/cm}^2$) → $L/\lambda \leq 0.1$: standard IB invalid. Less ‘region’ for absorption, but higher densities → **higher collision rates**.

→ **Short pulse, high intensities**: plasma becomes hot enough that collisional effect become small
  
  ↓
  
  nonlinear collisionless absorption.
Collisional Absorption

Once the plasma electrons are accelerated by the laser electromagnetic field, they can undergo Coulomb collisions with other electrons or ions, so that initial ordered motion acquires stochastic features and part of the radiation energy is transferred to the plasma. While electron-electron collisions contribute only to thermalization of the distribution function, and ion-ion collisions occur on longer timescales than that of interest, the absorbed energy is almost exclusively due to electron-ion collision.
Collisional Absorption

Collisional effects are usually introduction to the plasma by adding damping term, proportional to the particle velocity, within Lorentz equation of motion:

$$m_e \frac{d\vec{v}_e}{dt} = -e \left( \vec{E} + \frac{1}{c} \vec{v}_e \times \vec{B} \right) - m_e \nu_{ei} \nu_e,$$

where $\nu_{ei}$ is the electron-ion collision frequency, physically arises from binary collisions, resulting in a frictional drag on the electron motion.
Collisional Absorption

Collision frequency

\[
\nu_{ei} = \frac{\sqrt{32\pi}}{3} \frac{n_e Z e^4}{m_e^2 v_{te}^3} \ln(\Lambda) \approx 2.91 \times 10^{-6} Z n_e (T_e [eV])^{-3/2} \ln(\Lambda)
\] (5.5)

- \(Z\) - number of free electrons per atom,
- \(n_e\) - Electron density in cm\(^3\),
- \(T_e\) - electron temperature,
- \(\ln(\Lambda)\) - Coulomb logarithm, with usual limits, \(b_{min}\) and \(b_{max}\), of electron-ion scattering cross section.

\[
\Lambda = \frac{b_{max}}{b_{min}} = \lambda_{De} \frac{k_B T_e}{Z e^2} = \frac{9 N_{De}}{Z},
\] (5.6)

where \(
\lambda_{De} = \left(\frac{k_B T_e}{4\pi n_e e^2}\right)^{1/2} = \frac{v_{te}}{\omega_p}
\) and \(N_{De} = \frac{4\pi}{3} \lambda_{De}^3 n_e\) in number of particles in Debye sphere.
Collisional Absorption

Equation (5.5) shows that as the electron temperature rises, the collision rate decreases.

In Fig. 5.1, $\nu_{ei}$ is depicted as a function of the temperature for a solid density plasma. The value decreases below the 1% of the laser frequency ($\gamma_0 = c/\lambda$ for Ti-Sapphire laser) at $T_e = 10$ keV, which can be easily obtained during interaction of intense lasers with solids.

![Graph showing the relationship between $\nu_{ei}/\nu_0$ and $T_e$ (keV)]

**Fig. 5.1:** Electron-ion collisional frequency (electromagnetic terms of laser frequency $\nu_0$) vs electron temperature ($T_e$). $\nu_{ei}$ is evaluated from Eq. (5.5) for a solid density plasma ($n_e = 10^{23}$ cm$^{-3}$) with $z = 1$ and $\lambda_L = 0.6$ μm.
Collisional Absorption

Energy absorbed by a plasma due to electron-ion collisions, can be evaluated by mean of Helmholtz equations of wave propagation.

The coupling of classical Spitzer heat flow equation with collisional absorption by a sharp density profile leads to a plasma surface temperature scaling as:

$$T_e \sim (n_e Z)^{1/12} I_a^{1/3} t^{1/6},$$

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$I_a$ - absorbed intensity,  
$t$ - time coordinate.

→ The above mentioned relation the fact that collisional absorption is ruled out before the ultra-intense laser peak reaches the plasma.  
→ Collisional frequency $\nu_{ei}$ is also affected by the fact that at high irradiance, the electron quiver velocity becomes comparable to electron thermal velocity.
Collisionless Absorption

Other absorption mechanism that can couple laser energy to a hot, solid-density target?

→ Resonance absorption (long density scale lengths) – Denisov (1937)

→ Anomalous (collisionless) skin effect – Weibel (1967)

→ ‘Vacuum heating’ – Brunel (1987)

→ Relativistic $\vec{J} \times \vec{B}$ heating
Resonance Absorption

The conditions required to trigger resonant absorption are a longitudinal component of the laser optical field and a mild density gradient of the plasma, so that

$$\overrightarrow{E} \cdot \nabla n_e \neq 0$$
Resonance Absorption

Configuration as shown in fig. 5.2. electromagnetic field reflects at $n_e = n_{cr}(\cos \theta)^2$, where $\theta$ is the incidence angle. $p$-polarized light wave tunnels through the critical surface ($n_e = n_{cr}$).

At $n_e = n_{cr}$, the longitudinal component of the electric field oscillates at the plasma frequency and couples with the electrons exciting a plasma wave.

Plasma wave grows resonantly over some laser periods to be damped with consequent heating of plasma by collisions, Landau damping or, at higher intensities, by wave breaking.

Numerical simulations of laser–matter interaction demonstrated:

→ resonance absorption leads to a Maxwellian ‘tail’ of hot electrons, the temperature of hot electrons grow roughly as $(I\lambda^2)^{1/3}$.

→ resonance absorption is efficient for long pulses (>$\text{ps}$) and large plasma scale-lengths (>\text{µm}).
Vacuum Heating (Brunel Mechanism)

In a step profile, the longitudinal oscillations of electrons go across a step density gradient, with an amplitude of $\approx eE_L/m_e \omega^2 = \nu_{osc}/\omega$. Resonance breaks down if this amplitude exceeds the density scale length $L$ i.e.

$$\text{if } \nu_{osc}/\omega > L.$$ 

Thus

→ In Brunel Mechanism collisionless process is characterized by a step like plasma profile and a strong longitudinal field.

→ In this configuration, the electric field of the incident laser can drag electrons from plasma surface directly into vacuum.

→ Laser field reverses its direction and same electron will be turned around and accelerated back into the plasma.

→ Once such electrons, carrying an energy proportional to the square of quiver velocity, overcome the skin depth $c/\omega_{pe}$, they do not experience the laser field anymore, and transport their energy inside the plasma.

→ This results in a pulsed generation of fast electron bunches directed into the
Vacuum Heating (Brunel Mechanism)

Brunel proposed a one-dimensional ‘capacitor model’ where target is represented by a perfect conductor, localized in the $x > 0$ region, while in the vacuum ($x > 0$) an oscillating electric field proportional to laser amplitude can extract electrons from the target and re-inject them back.

Fig. 5.3: Capacitor model of Brunel heating mechanism.
Vacuum Heating (Brunel Mechanism)

Driving Electric field,

$$E_d = 2E_L \sin \theta$$  \hspace{1cm} (5.8)

Driving field pulls a sheet of electron out to a $\Delta x$ distance from its initial position.

Surface number density of sheet, $\Sigma = n_e \Delta x$

Electric field (between $x = -\Delta x$ and 0), $E = 4\pi e \Sigma$

By equating this field with driving field

$$\Sigma = \frac{2E_L \sin \theta}{4\pi e}$$  \hspace{1cm} (5.9)

Charge sheet acquires velocity $v_d = 2v_{osc} \sin \theta$, while returns to original position.

The average energy density absorbed per laser cycle (assuming all electrons lost to solid)

$$P_a = \frac{\Sigma m_e v_d^2}{\tau} = \frac{1}{16\pi^2} \frac{e}{m_e \omega} E_d^3$$  \hspace{1cm} (5.10)
Vacuum Heating (Brunel Mechanism)

Comparing the absorbed energy density with incoming laser power:

\[ P_L = cE_L^2 \cos \theta / 8\pi \]

**Fractional Absorption Rate**

\[ \eta_a = \frac{P_a}{P_L} = \frac{4}{\pi} \frac{a_0}{\cos \theta} \frac{(\sin \theta)^3}{c} \]

where \( a_0 = \nu_{osc}/c \)

Two corrections are given here to improve the model:

→ Taking into account the effect of reduced driver field amplitude due to absorption,

\[ E_d = \left[ 1 + (1 - \eta_a)^{1/2} \right] E_L \sin \theta \]

and

→ Relativistic return velocities: \( U_K = (\gamma - 1)m_e c^2 \)
Incorporating above corrections to fractional absorption

\[ \eta_B = \frac{1}{\pi a_0} f \left[ (1 + f^2 a_0^2 (\sin \theta)^2)^{1/2} - 1 \right] \frac{\sin \theta}{\cos \theta}. \]  

(5.12)

where \( f = 1 + (1 - \eta_a)^{1/2} \)

In strong relativistic regime \( f a_0 \sin \theta \gg 1 \),
we obtain \( f = 2 \left( \frac{\alpha'}{\pi} + 1 \right)^{-1} \) such that:

\[ \eta_{rel,0} = \frac{4\pi \alpha'}{\left( \alpha' + \pi \right)^2}. \]  

(5.13)

where \( \alpha' = (\sin \theta)^2 / \cos \theta \)

Fig. 5.4: Angular dependence of vacuum heating by Brunel model.
Vacuum Heating (Brunel Mechanism)

From the description of Brunel Model to temperature (mean kinetic energy) of the accelerated electrons can be estimated by:

\[
T_{h,B} [\text{keV}] = 2m_e \nu_{osc}^2 (\sin \theta)^2 \cong 3.7 \times 10^{-16} \left( I \lambda^2 \left[ \frac{W \mu m^2}{cm^2} \right] \right), \tag{5.14}
\]

The capacitor model is not self-consistent, since a zero field in the target region would require a surface charge density which is artificial.

If the laser impinges normally on the plasma surface (or if no longitudinal component of electric field is available), Brunel mechanism is not activated.
\[ \vec{J} \times \vec{B} \text{ Heating} \]

In intense laser – matter interaction the nonlinear effect which resulted from the coupling of the \( \vec{B} \) – component of wave with the current of electrons are oscillating along the transverse \( \vec{E} \) field, namely the “\( \vec{J} \times \vec{B} \)” acquires importance.

\( \vec{J} \times \vec{B} \) coupling provides a longitudinal force which can transfer the radiation energy to the plasma electrons similarly to the electric field force in Brunel Model.

If a plane, elliptically polarized wave of eccentricity \( 0 < \varepsilon < 1 \) propagates along the \( x \) direction, then the vector potential can be written as,

\[ \hat{A} = \frac{A(x)}{\sqrt{1 + \varepsilon^2}} (\hat{y} \cos \omega t + \varepsilon \hat{z} \sin \omega t) \quad (5.15) \]

where \( \hat{y} \) and \( \hat{z} \) are the transverse direction unit vectors.
\[ \vec{J} \times \vec{B} \text{ Heating} \]

The longitudinal force (resulting from the $\vec{J} \times \vec{B}$ coupling) can be written as:

\[ \frac{e}{c} \vec{v} \times \vec{B} = \hat{x} \frac{e^2 \partial A^2(x)/\partial x}{4 m_e \gamma c^2} \left( 1 + \frac{1 - \varepsilon^2}{1 + \varepsilon^2 \cos 2\omega t} \right) \]  \hspace{1cm} (5.16)

The first term on the RHS is the usual steady part which pushes on density profile. The second term is a high frequency oscillating component which leads to heating. The latter can potentially inject bunches of electrons beyond the skin depth, similar to Brunel Model, but with a doubled frequency.
Anomalous Skin Effect

As stated before (Eq. (5.5)), when the temperature is increased $\nu_{ei}$ becomes negligible as the thermal velocity $\nu_{te}$ grows. In this case the electrons can oscillate in the skin layer without occurring in collisions and tow regimes of absorption are pointed out.

$\rightarrow$ if $\lambda_{te} \gg l_s$: the electron population sufficiently agitated to carry the field energy away from the skin layer before a pulsation of the wave is completed. Thus the laser field influence is transmitted beyond the skin depth and thermalization becomes non-local, in a process named Anomalous Skin Effect.

$\rightarrow$ if $\lambda_{te} \ll l_s$: electrons can perform several oscillations in the laser field, within the skin depth. While this motion is active, they can receive a nonadiabatic thrust by the laser ponderomotive potential or by ambi-polar potential arising via charge separation at the plasma boundary. This mechanism is named Sheath Inverse Bremsstrahlung, since electron oscillations are affected by 'collisions' against a sheath potential.
Hot Electron Generation

The intense laser interaction with overdense plasma

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Energy carried by the electromagnetic pulse efficiently absorbed at the critical density surface \( n_e = n_c \)

↓

Multiple phenomena (e.g. collisionless absorption)

↓

Collisionless Absorption of laser field

↓

Absorbed energy distributes to the plasma electrons.
This the electron gain hundred of keVs – several MeVs of KE.

↓

Electron population acquires a bi-Maxwellian (two temperature Maxwellian) spectrum with characteristic temperature \( T_e \) and \( T_h \).
Hot Electron Generation

A fraction of every electrons, typically less than the 1% is heated up to extremely high (keV-MeV) temperatures, while the rest of the plasma electrons acquire a relatively lower mean energy. These super thermal electrons are referred as hot electrons or fast electrons.

Fig. 5.5: Bi-Maxwellian electron energy spectra (obtained by numerical simulation). The distribution approximated of two Maxwellian curves at different temperatures.
Long pulses

Energy balance:
Absorbed laser flux carried by a population of free-streaming hot electron (with temperature $T_h$):

$$\eta_a I_0 \equiv n_h \nu_h T_h$$  \hspace{1cm} (5.17)

Pressure Balance:

$$\eta_e = \frac{I_0}{c k_B T_e} \approx 200 \frac{I_{18} \lambda_u^2}{T_{keV}} n_c$$  \hspace{1cm} (5.18)

From above conditions,

$$T_h \approx 14 (I_{16} \lambda_u^2)^{1/3} T_e^{1/3} \text{ keV}$$  \hspace{1cm} (5.19)
Short pulse Scaling

→ Brunel Model:

\[ T_{h,B} = \frac{1}{2} m v_d^2 \approx 3.7 I_{16} \lambda_u^2 \] \hspace{1cm} (5.20)

→ Electromagnetic PIC simulations:

\[ T_{h,GB} \approx 7 (I_{16} \lambda_u^2)^{1/3} \] \hspace{1cm} (5.21)

→ Relativistic model (ponderomotive scaling):

\[ T_{h,W} \approx m_e c^2 (\gamma - 1) \approx 511 \left[ (1 + 0.73 I_{18} \lambda_u^2)^{1/2} - 1 \right] \text{ keV} \] \hspace{1cm} (5.22)
Problems

Q5.1. True or False. Inverse Bremsstrahlung is the process in which an electron absorbs a photon while colliding with an ion or with another electron.
A5.1. True.

Q5.2. Collisional Absorption of laser light will be more effective for long wavelength pulse or short wavelength since the absorption rate is proportional to plasma density and inversely dependent on electron temperature.
A5.2. Short wavelength.

Q5.3. Why collisional effect in plasma ruled out at high irradiance of laser field?
A5.3. The collision frequency and the IB rate drop with increasing laser intensity and / or increasing $T_e$. 
Problems

Q5.4. Where does absorption comes from?
A5.4. According to Poynting’s Theorem – the net energy absorption per unit volume and per cycle is given by \( \langle \mathbf{J} \cdot \mathbf{E} \rangle \), the phase shift between \( \mathbf{J} \) and \( \mathbf{E} \). In absence of collision absorption cab be due to mode conversion and kinetic effects.

Q5.5. Write down the condition when collective plasma oscillation occurs in resonance absorption.
A5.5. Plasma Frequency = Wave Frequency.

Q5.6. True or False. In Vacuum heating, electrons cross the plasma boundary and return with high velocity.
A5.6. True.
Problems

Q5.7. Point out the absorption mechanism, when
(a) $v_{osc} > v_{th}$ and (b) $v_{osc} < v_{th}$.

A5.7. (a) Vacuum heating (b) Skin absorption.

Q5.8. Calculate the energy per electron according to ponderomotive scaling for normalised laser field, $a_0=8.5$.

A5.8. 3.8 MeV.

Q5.9. What do you mean by Bi-Maxwellian distribution of electrons?

A5.9. The combined distribution of two Maxwellian corresponding to different temperature electrons.

Q5.10. Why at high intensities ($>10^{15}\text{W/cm}^2$), energy transfer between laser and particles are not be explained by electron-ion collisions?

A5.10. first the electron quiver velocity may exceed than the thermal velocity and second the collision frequency scales as $(k_B T_e)^{-3/2}$. 

